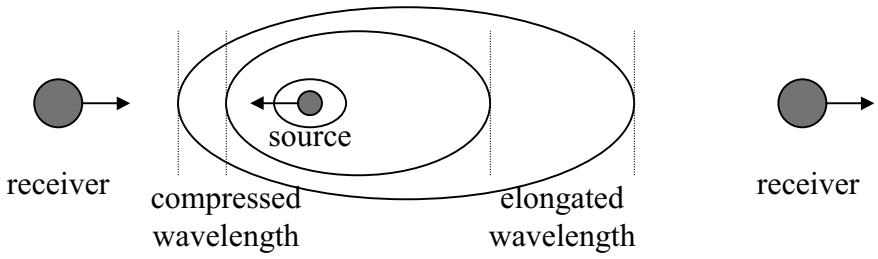
Chapter 8 : Color and Spectral Doppler

I. Doppler Effect

- In addition to imaging anatomical structures in the body, sound waves can also be used to detect moving objects using Doppler. The most important clinical application of Doppler ultrasound is blood flow measurement. The basic Doppler principles are shown in the following figure.



- When both the source and the receiver are stationary, the frequency experienced by the receiver is simply $f_s = c/\lambda$, where c is the speed of sound and λ is the wavelength. Suppose the source is stationary and the receiver is moving toward the source at a velocity of v_r, the listener will experience $(cT/\lambda + v_r T/\lambda)$ waves in time T. Therefore, the frequency experienced by the receiver is $f = (c + v_r)/\lambda$ and the Doppler shift frequency is $f_d = f - f_s = v_r f_s / c$. Note that generally v_r can also be a negative number representing the receiver moving away from the source.

- When the source is moving and the receiver is stationary, the Doppler effect is different from the above situation. When the source moves toward the stationary listener, the wavelengths are in effect shortened because between emission of two consecutive waves, the source has moved. Suppose the source velocity is v_s, the wavelength is compressed from c/f_s to $(c - v_s)/f_s$. Hence, the frequency experienced by the receiver becomes $f = f_s c/(c - v_s)$ and the Doppler shift frequency is $f_d = f - f_s = f_s v_s/(c - v_s)$. Similar to the previous situation, v_s can also be a negative number indicating the source moving away from the receiver.

- Note that it is not just the relative motion between source and receiver that is important, which one is in motion also needs to be considered for sound waves. This is different from Doppler shifts for light waves, in which it is impossible to identify a medium of transmission relative to which the source and the receiver are moving. Therefore, only the relative motion between the source and the receiver is distinguishable for light waves.

- If both the source and the receiver are moving, the Doppler shift frequency becomes

$$f_d = f_s (v_r + v_s)/(c - v_s) .$$

  Assuming the source velocity is much smaller the sound velocity (i.e., $v_s << c$), the above equation can be simplified as

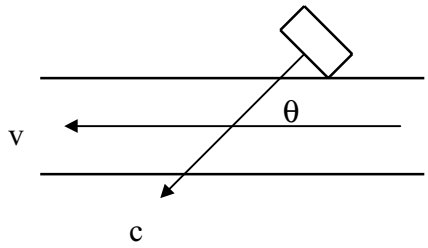
$$f_d \approx f_s (v_r + v_s)/c .$$

  This assumption is true for diagnostic ultrasound since the velocity of typical physiological flows in the body are at most at the order of a few meters per second, whereas sound velocity in blood is around 1500 meters per second.

- Since motion of the source relative to the receiver causes a change in the observed sound frequency, blood flow velocity can be measured by detecting Doppler frequency shift of echoes backscattered from moving blood. The primary scattering site in the blood that produces echoes is the red blood cell. The scattering cross section of the red blood cell is about 1000 times larger than that of the platelet. Leukocytes are not present in sufficient numbers to influence the total backscattered signal.

- Similar to the previous discussion on ultrasound speckle, the incident ultrasonic pulse also extends over many red blood cells (there are a few million red blood cells per mm^3 and they are not resolvable by typical ultrasonic imaging systems). Therefore, the signals reflected from the red blood cells within a sample volume are generally not in phase. However, they do vary in unison, i.e., they do change in an orderly fashion such that their velocity can still be estimated by using the Doppler effect.

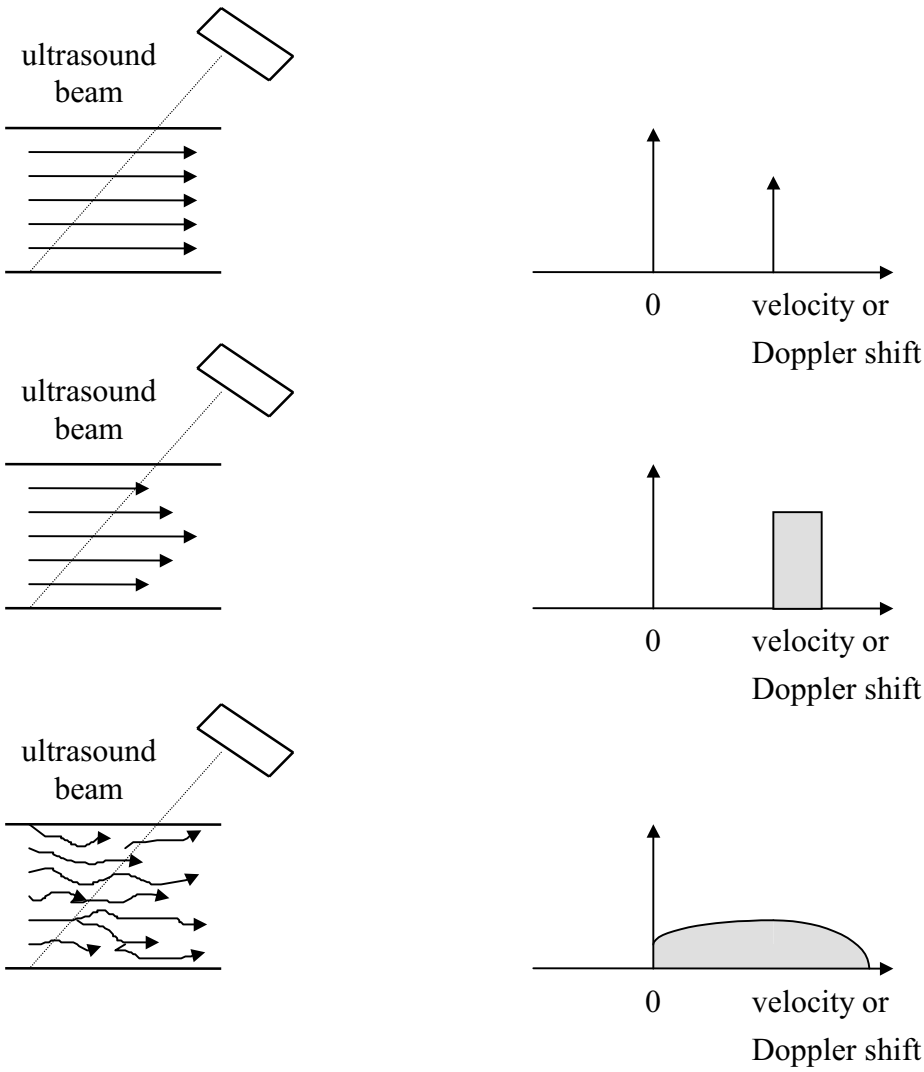
- Considering the following drawing, the basic Doppler equation can be written as



$$f_d = \left(\frac{2vf_s}{c}\right)\cos\theta, \ (v << c),$$

where v is the flow velocity, c is the propagation velocity of sound waves, f_d is the detected Doppler frequency shift, f_s is the source frequency and θ is the angle between the flow and the ultrasound beam. Note that the frequency shift is doubled due to round trip propagation and only flows parallel to the ultrasound beam can be detected.

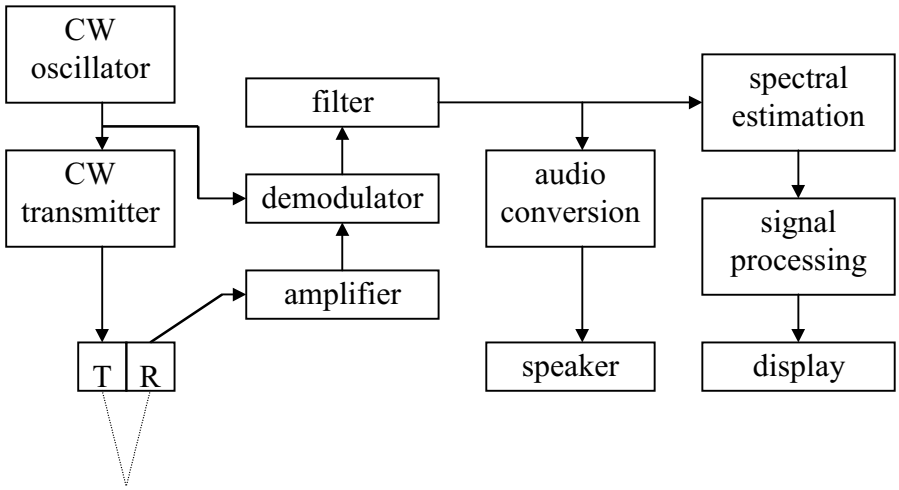
- Flow patterns and their corresponding velocity profiles can be illustrated by the following examples:



- Frequency shifts due to Doppler effects are typically estimated by short-time Fourier transform or efficient autocorrelation techniques. The former is usually called Spectral Doppler, in which only flows along a single ultrasound line (continuous wave, CW) or flows within a particular sample volume (pulsed wave, PW) are estimated. The latter only estimates simple flow parameters, such as mean velocity, flow energy and velocity variance, and is capable of displaying two-dimensional flow information in real-time. Since flow parameters (velocity, variance and energy) are mapped to different colors, this is also known as the Color Doppler mode.

II. CW (Continuous Wave) Doppler

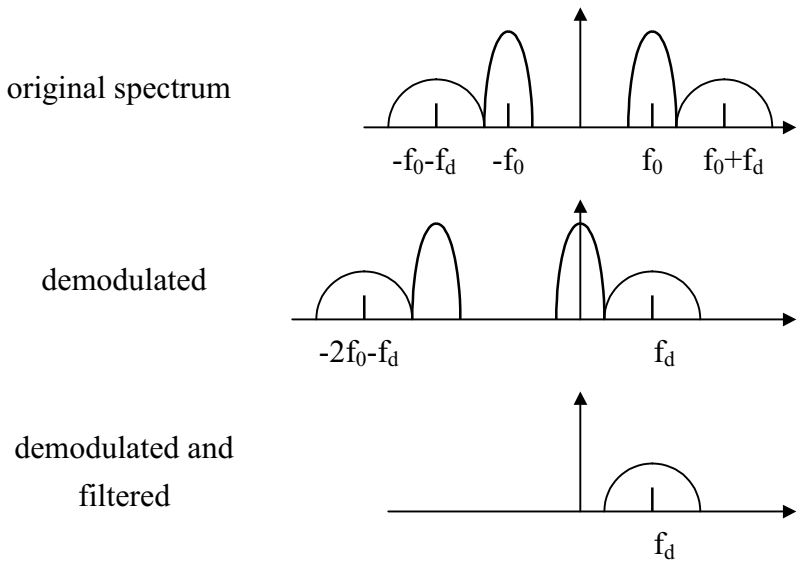
- The first reported application of the Doppler effect in medical ultrasound was CW (continuous wave) velocity measurement of blood. A basic CW Doppler instrument is illustrated in the following drawing.



- In CW mode, one half of the transducer array is continuously transmitting and the other half is continuously receiving. Alternatively, a special transducer made with two piezoelectric crystals for transmitting and receiving the ultrasonic wave can be used. The former is known as array CW and the latter is known as AUX (auxiliary) CW. Note that the AUX CW transducer is only for Doppler measurement and is not suitable for two-dimensional imaging.

- Since one half of the transducer is continuously transmitting, range information cannot be extracted from the returning echoes. In other words, flow information along the entire ultrasound beam is measured by the receiver. Additionally, the frequency downshift due to attenuation can be ignored since the ultrasonic wave is a very narrow band signal (close to zero bandwidth).

- The Doppler shift frequencies are extracted by the following steps. First, the original signal is demodulated down to baseband. Second, the demodulated signal is low pass filtered to filter out higher frequency components. Third, a high pass filter (a.k.a. wall filter) is used to filter out non-moving and low velocity signals from stationary tissue and relatively slow moving vessel walls. These signals are generally much stronger than the flow signals. The resultant Doppler spectrum is then estimated and displayed. The frequency domain representation of this process can be illustrated by the following drawing. It is worth mentioning that returning echoes from blood are generally highly non-stationary. Therefore, extraction of flow information is indeed a time-frequency analysis.



- The first spectral displays were time-interval histograms made by measuring the times between zero crossings of the Doppler shift waveform. The time intervals are then lumped into bins and displayed as a histogram that gets updated many times in a heart cycle. Modern ultrasonic imaging systems typically use short time Fourier transform. In this case, signal in a moving window (in time) is Fourier transformed and the resultant spectrum represents the velocity distribution of

blood flow at that particular instance. It is assumed that the signal within the window of analysis is stationary and the Fourier transform is typically done by 32-128 points FFT (Fast Fourier Transform).

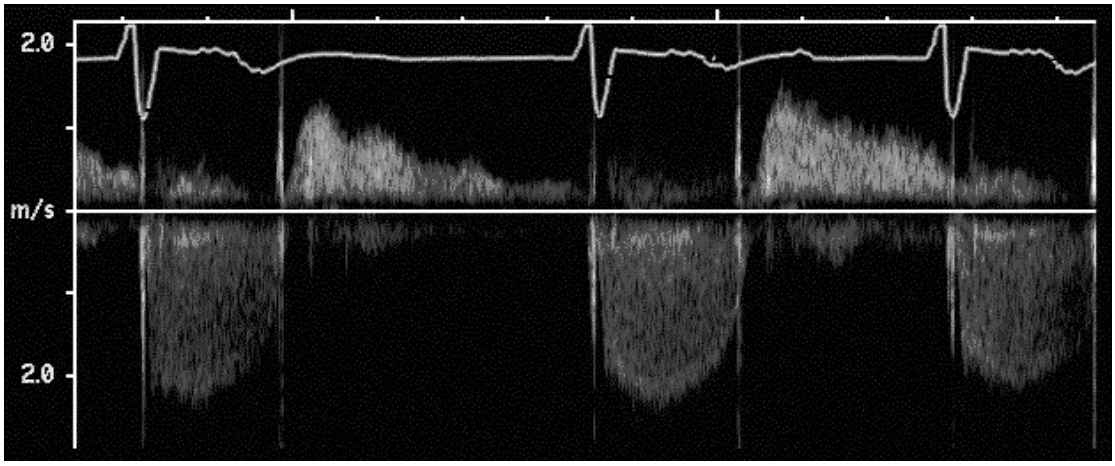
- Other time-frequency analysis techniques, such as AR (auto regressive), wavelet and adaptive cone kernel techniques, have also been explored. However, if only the general features of the flow distribution are of interest, the short time Fourier transform is the most stable and interpretable. Other techniques, in particular model based techniques, tend to break down when the model does not fully characterize the flow.

- Typically only magnitudes of the estimated spectra are displayed and the magnitudes are generally converted to a logarithmic scale. In addition, filtering may be applied in both the frequency direction and the temporal direction prior to display.

- For blood velocities present in the body and for frequencies typically used by the ultrasonic imaging systems, the Doppler shift frequencies from blood happen to occur in the human audible range (near DC to 20KHz). In fact, early CW instruments simply played the Doppler shift frequencies into a speaker. Modern CW instruments differentiate positive frequency shifts from negative frequency shifts. Positive ones, which correspond to flows moving toward the transducer, are played in one channel of a stereo pair of speakers and negative ones, which correspond to flows moving away from the transducer, are played in the other channel. This mode is also known as Audio Doppler.

- Since human ear is very adept at recognizing frequencies and bandwidths of signals in the presence of background white noise, Audio Doppler can offer great diagnostic information once the operators were trained well enough to recognize the normal Doppler shift sounds from abnormal flow patterns. It is not unusual that weak flows are "heard" through Audio Doppler but are hardly "seen" in the CW Doppler strip.

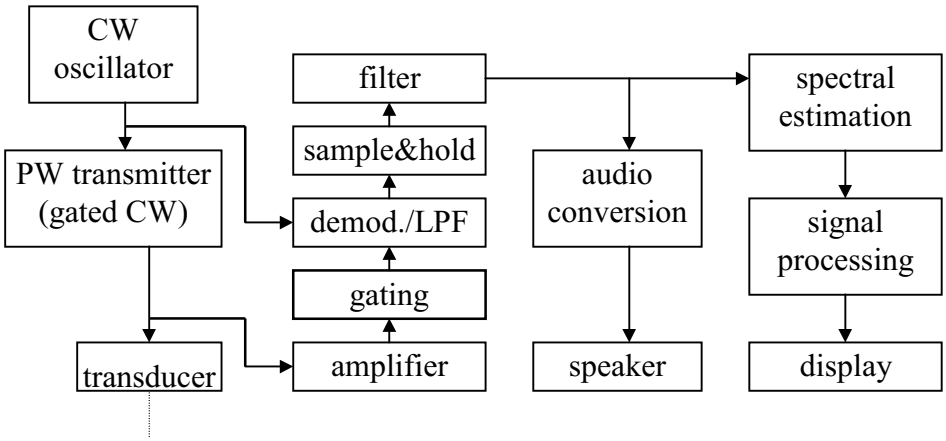
- A typical CW Doppler strip is shown in below.



III. PW (Pulsed Wave) Doppler

- The major limitation of CW Doppler is that it is sensitive to blood flows along the entire length of the ultrasonic beam and the position of individual flows cannot be localized in range. To overcome this problem, pulsed Doppler was developed.

- A basic PW Doppler instrument is illustrated in the following drawing.



- One of the main differences between CW Doppler and PW Doppler is that PW Doppler transmits a short sinusoid burst, instead of a continuous sine wave. By gating the received signals to correspond to the pulse's time of flight to the point of interest, one can interrogate a small sample volume instead of the entire length of the ultrasonic beam.

- Unlike CW Doppler, frequency (i.e., velocity) aliasing may occur in PW Doppler.

This is due to the fact that the returned echoes are sampled at a fixed time after the burst is transmitted. This time interval is also known as PRI (pulse repetition interval). According to the Nyquist criterion, the maximum Doppler shift frequency ($f_{\max}$) that can be detected for a given PRI is

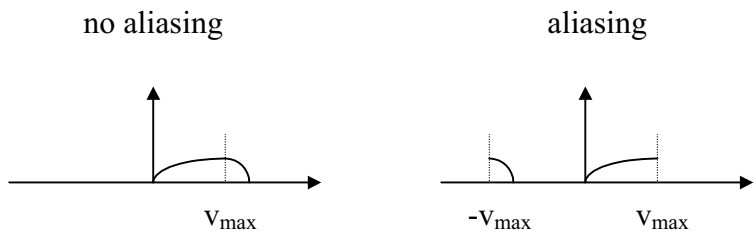
$$2f_{\max} \le \frac{1}{PRI}.$$

Therefore, the maximum velocity ($v_{\max}$) of the flow projected along the direction of the ultrasonic beam that can be detected without aliasing is

$$2f_{\max} = \frac{4v_{\max}f_s}{c} \le \frac{1}{PRI}$$
$$v_{\max} \le \frac{\lambda}{4 \cdot PRI}$$

where λ is the wavelength at the carrier frequency (i.e., without Doppler shift).

- Due to the sampling requirement, a trade-off exists between the maximum sample depth and maximum un-aliased blood velocity. When aliasing occurs, the high velocity components (higher than $v_{\max}$) show up in the other side of the spectrum. This is illustrated as follows.

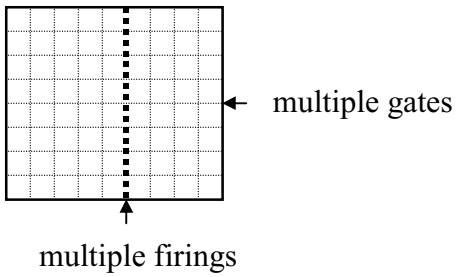


- In addition to spectral aliasing, PW Doppler is also susceptible to range ambiguity. In other words, signals from sample volumes which correspond to an integer multiple of $c \cdot PRI / 2$ away from the main sample volume may also contribute to the real Doppler signal.

- The CW spectral estimation and signal processing techniques are also applicable for PW. Similarly, the signal can be converted to the Audio Doppler format and sent to the speakers. Unlike CW Doppler, however, the transducer array does not have to be divided into two parts for continuous transmission and reception.

IV. Color Doppler

- Spectral Doppler provides flow information along a fixed direction (CW) or within a single Doppler gate (PW). Color Doppler, on the other hand, provides real-time two-dimensional flow information. The ultrasonic data acquisition process in Color Doppler is similar to that in B-mode except that the ultrasonic pulse needs to be fired multiple times along the same direction sequentially before moving to the next beam. In other words, there are many Doppler gates (similar to the PW gate) along each ultrasonic beam and the flow profile within each gate is estimated. The detected flow information is then encoded in colors and superimposed on the two-dimensional B-mode image.



- In order to achieve sufficient frame rate for real-time imaging, the number of samples (i.e., the number of firings) used for flow estimation is typically limited to around 5 to 15. Compared to Spectral Doppler, where usually 32 to 128 point FFT is performed, the accuracy of flow estimation is limited and hence it is relatively a more qualitative tool for flow assessment. Spectral Doppler is often required for quantitative flow measurements.

- Similar to PW Doppler, the maximum flow velocity that can be detected without being aliased is limited to $\lambda/(4 \cdot PRI)$. Since there are multiple gates along an ultrasonic beam, the PRI effectively limits the maximum display depth. In other words, the maximum display depth cannot exceed $c \cdot PRI/2$, where c is the sound velocity.

- Instead of the previously described time-frequency analysis techniques (such as Fourier transform), more efficient flow estimation methods need to be employed in order to provide important aspects of blood flow with a limited number of samples. These important aspects include flow direction, mean velocity, flow

turbulence and flow energy. In the following, an autocorrelation technique capable of estimating the above parameters will be described.

- Let $S(t)$ be the signal received from a particular gate, its autocorrelation function $R(t)$ is defined as

$$R(t) \equiv \int_{-\infty}^{\infty} S(t+\tau) S^*(\tau) d\tau$$

where * denotes the complex conjugate. In addition, the power spectrum $P(\omega)$ is the Fourier transform of $R(t)$ based on the Wiener-Khinchine's theorem, i.e.,

$$R(t) = \int_{-\infty}^{\infty} P(\omega) e^{j\omega t} d\omega$$

where $\omega = 2\pi f$ and f denotes the frequency. It then follows that the mean frequency (i.e., the first order moment) can be represented by

$$\overline{\omega} = \frac{\int_{-\infty}^{\infty} \omega P(\omega) d\omega}{\int_{-\infty}^{\infty} P(\omega) d\omega}.$$

Since

$$R(0) = \int_{-\infty}^{\infty} P(\omega) d\omega$$

and

$$-j \frac{dR(t)}{dt}\Big|_{t=0} = \int_{-\infty}^{\infty} \omega P(\omega) d\omega,$$

we have

$$\overline{\omega} = -j \frac{R'(0)}{R(0)}$$

where ' represents the time derivative. The above equation indicates that the mean Doppler shift frequency can be directly estimated using time domain signals without calculating the frequency spectrum.

- The above equation can be further simplified. Let $R(t) = |R(t)| e^{j\theta(t)}$, it is straightforward to see that $|R(t)|$ is an even function and $\theta(t)$ is an odd function. Let $A(t) \equiv |R(t)|$, we have

$$R'(t) = A'(t) e^{j\theta(t)} + j\theta'(t) A(t) e^{j\theta(t)}.$$

Hence,

$$R'(0) = jA(0)\theta'(0)$$

and

$$R(0) = A(0).$$

Using the above relations, the mean frequency $\overline{\omega}$ can be simplified as

$$\overline{\omega} = \theta'(0) \approx \frac{\theta(T) - \theta(0)}{T} = \frac{\theta(T)}{T},$$

where T is the time interval between two consecutive ultrasonic pulses (i.e., PRI). Note that $\theta(0) = 0$ since $\theta(t)$ is an odd function.

- In practice, $R(T)$ is simply the discrete autocorrelation function evaluated at the first lag. Since the number of firings per ultrasonic beam is limited, $R(T)$ is obtained by summing over a limited range. Suppose there are only two samples available (i.e., $S(0)$ and $S(T)$), $R(T) = S(T)S^*(0)$ and hence $\theta(T) = \angle S(T) - \angle S(0)$. In this case, the Doppler shift frequency is simply the phase change between two consecutive measurements divided by time T. Since the phase is related to the distance between the object and the transducer, the mean Doppler shift frequency also represents the change in distance during time T, which is indeed the definition of velocity.

- The direction of the flow is determined by the sign of the mean frequency. A positive mean frequency means a flow is moving toward the transducer, a negative mean frequency means a flow is moving away from the transducer.

- The extent of turbulence in blood flow can be related to the variance of the power spectrum. In other words, the spectrum spread broadens in the presence of flow turbulence. The variance σ^2 is represented as the following:

$$\sigma^2 = \frac{\int_{-\infty}^{\infty} (\omega - \overline{\omega})^2 P(\omega)\, d\omega}{\int_{-\infty}^{\infty} P(\omega)\, d\omega} = \overline{\omega^2} - \overline{\omega}^2.$$

Since

$$-\frac{d^2 R(t)}{dt^2}\Big|_{t=0} \equiv -R''(0) = \int_{-\infty}^{\infty} \omega^2 P(\omega)\, d\omega,$$

we have

$$\sigma^2 = -\frac{R''(0)}{R(0)} + \left(\frac{R'(0)}{R(0)}\right)^2.$$

- The variance can also be simplified by noting that

$$R''(0) = A''(0) - A(0)(\theta'(0))^2.$$

Therefore,

$$\sigma^2 = (\theta'(0))^2 - \frac{A''(0)}{A(0)} - (\theta'(0))^2 = -\frac{A''(0)}{A(0)}.$$

Expand $A(t)$ in a Taylor series, we obtain

$$A(t) = A(0) + \frac{t^2}{2}A''(0) + R$$

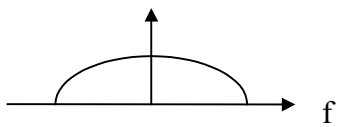
where R is the remainder. Using the above equations, we have

$$\sigma^2 \approx \frac{2}{T^2}\left(1 - \frac{A(T)}{A(0)}\right) = \frac{2}{T^2}\left(1 - \frac{|R(T)|}{R(0)}\right).$$

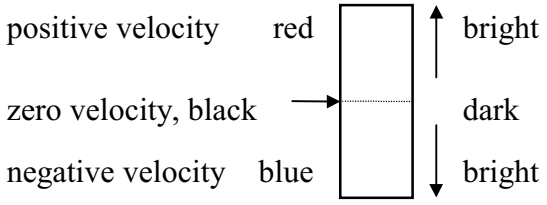
The flow turbulence can also be estimated directly from the received time domain signals.

- The total energy E of the flow signal can be found by integrating the power spectrum over the entire frequency range, i.e., $E = \int_{-\infty}^{\infty} P(\omega)\,d\omega$. Since $\int_{-\infty}^{\infty} P(\omega)\,d\omega = R(0)$, the parameter E can also be estimated directly in time domain.

- Consider the following hypothetical flow, the mean frequency $\overline{\omega}$ becomes zero due to the symmetry. The energy E, however, is non-zero since it represents the total area under the spectrum. Therefore, the energy mode is sometimes preferred in some clinical situations, particularly when the direction of the flow is not of primary interest.

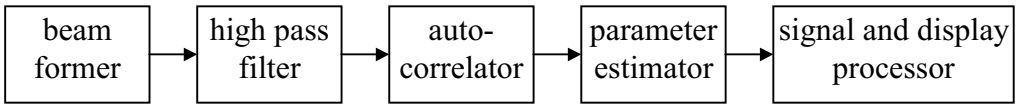


- As mentioned previously, these flow parameters are encoded in colors for the real-time display. The following example shows a typical way of mapping velocities (including directionality) to different colors.



- Two-dimensional maps can also be constructed to encode two different parameters into colors simultaneously. For example, one dimension may represent the velocity and the other may represent the variance.

- A simple block diagram for a Color Doppler processing chain is shown below. Note that the purpose of the high pass filter it the same as that in Spectral Doppler, i.e., it is a wall filter to filter out strong signals from non-moving and slow moving structures.



- Since multiple firings along an ultrasonic beam are required for flow estimation, the frame rate impact becomes a significant issue. In order to achieve sufficient frame rate to image the flow dynamics, Color Doppler processing is usually confined to a small area of interest. Outside this area, only B-mode signals (i.e., single firing) are acquired. In addition, the beam spacing may be chosen higher than the Nyquist criterion to gain frame rate at the price of aliasing artifacts. In some high end systems, parallel beams are formed simultaneously in order to further increase frame rate.

- Doppler shift frequencies can also be used to estimate other types of motion, such as heart motion. In this case, motion abnormalities can be visualized by using color encoded, two-dimensional Doppler image in real time. Since the signal from heart muscle is sufficiently strong, the design of the wall filter can be greatly simplified.

- One inherent limitation of motion estimation using Doppler effect is that only

motion parallel to the beam direction can be detected. Therefore, how to detect motion in two dimensions (or three dimensions, in general) is an important topic in diagnostic ultrasound.